CONTROL SYSTEM

→ LECTURE NOTE

→ OBJECTIVE QUESTION BANK

6th SEMESTER

ELECTRICAL BRANCH

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6th semester ELECTRICAL (1)

Objective Question & Ans. control system

1. What is control system?

Ans A system consist of a number of components connected together to perform a specific function. In a system when the output quantity is controlled by varying the input quantity, then the system is called control system.

2. What are the two major types of control system?

Ans The two major type of control system are open loop and closed loop.

3. Define open loop control system?

Ans→ The control system in which the output quantity has no effect upon the input quantity is called open loop control system. The output is not feedback to input.

4. Define closed loop control system?

Ans The control system in which the output has an effect upon the input quantity so as to maintain the desired output value is called closed loop system.

5. Define transfer function?

Ans The transfer function of a system is defined as the ratio of Laplace transform of output to the Laplace transform of input with zero inital condition.

6. What is block diagram?

Ans A block diagram of a system is a pictorial representation of the functions performed by each component of the system and shows the flow of signals.

7. What is signal flow graph?

Ans A signal flow graph is a diagram that represents a set of simultaneous algebric equations.

8. Write Mason's gain formula?

$$T.F = \frac{P_K \Delta_K}{\Delta}$$

P_K = forward path in the signal flow graph

Δ_K = 1 − [sum of all loop not touching forward path P_K]
+ [sum of product of two non touching loop not touching forward path]

Δ = 1 − [sum of all loop] + [sum of product of two non touching loop]

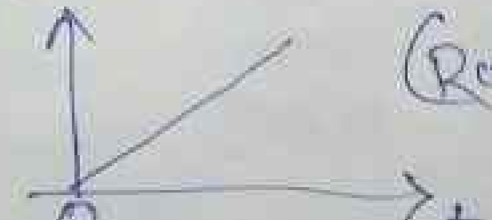
9. What is step signal?

Ans The step signal is a signal whose value ~~increases linearly~~ change from zero to "A" at $t = 0$ and remains constant at 'A' for $t > 0$

(step signal)

10. What is Ramp signal?

Ans The Ramp signal is a signal whose value increases linearly with time from an inital value of zero at $t = 0$.



(Ramp signal)

11. What is transient response?

Ans The transient response ~~of the system~~ is the response of the system when the system changes from one state to another.

(3)

12. What is steady state Response?
Ans The steady state response is the response of the system when it approaches to infinity.

13. Define Damping ratio?
Ans Damping ratio is defined as the ratio of actual damping to critical damping.

14. List the Time domain specification?
Ans The time domain specification are
(i) Delay time
(ii) Rise time
(iii) Peak time
(iv) Peak overshoot

15. Define Delay time?
Ans The time taken for response to reach 50% of the final value for the very first time is called delay time.

16. Define rise time?
Ans The time taken for the response to rise from 0% to 100% for the very first time is called rise time.

17. Define peak time?
Ans The time taken for the response to reach the peak value for the first time is called peak time.

18. Define Peak overshoot?
Ans Peak overshoot is defined as the ratio of maximum peak value measured from the maximum value to final value.

19. Define settling time?
Ans settling time is defined as the time taken by the response to reach and stay within specified error.

20. What is the need for a controller?

Ans The controller is provided to modify the error signal for better control action.

21. what are the different types of controller?

Ans (i) proportional controller
(ii) PI controller
(iii) PD controller
(iv) PID controller

22. what is proportional controller?

Ans It is a device that produces a control signal which is proportional to the input error signal.

23. What is PI controller?

Ans It is a device that produces a control signal consisting of two terms - one is proportional to the error signal and the other is proportional to the integral of error signal.

24. what is PD controller?

Ans → PD controller is proportional plus derivative controller which produces an output signal consisting of two times - one proportional to error signal and other proportional to the derivative of the signal.

25. what is the significance of Integral controller and derivative controller in a PID controller?

Ans The proportional controller stabilizes the gain but produces a steady state error. The integral controller reduces the steady state error.

26. What is the disadvantages of proportional controller? (5)

Ans The disadvantage of proportional controller is that it produces a constant steady state error.

27. What is the effect of PD controller on system performance?

Ans The effect of PD controller is to increase the damping ratio of the system and so peak overshoot is reduced.

28. What is the effect of PI controller on the system performance?

Ans The PI controller increases the order of the system by one, which results in reducing the steady state error.

29. What is steady state error?

Ans The steady state error is the value of the error signal e(t) when "t" tends to infinity (∞).

30. What are static error constant?

Ans Kp, Kv and Ka are called the static error constant.

31. What are the three constants associated with a steady state error?

Ans Positional error constant
Velocity error constant
Acceleration error constant

32. What are the main advantages of generalized error coefficient?

Ans Steady state is the function of time. Steady state can be determined from any type of input.

33. What is frequency response ? (6)

Ans The response of the system at the steady state when the input to the system is a sinusoidal signal is called as the frequency response.

34. What is Bandwidth ?

Ans The bandwidth is the range of frequency for which the system gain is more than 3dB. The bandwidth is a measure of the ability of a feedback system to reproduce the input signal noise reject.

35. Define Gain margin ?

Ans The gain margin is defined as the reciprocal of the magnitude of the open loop transfer function at the phase cross over frequency.

$$\text{Gain margin} = \frac{1}{|G(j\omega_{pc})|}$$

36. Define phase crossover frequency ?

Ans The frequency at which the phase of open loop transfer function crosses 180° is called phase cross over frequency.

37. What is Phase margin ?

Ans The phase margin is the amount of phase lag at the gain cross over frequency required to bring the system to the verge of stability.

38. Define gain cross over frequency ?

Ans The gain crossover frequency (ω_{gc}) is the frequency at which the magnitude of the open loop transfer function is unity.

39. What is Bode plot ?

Ans. The Bode plot is the frequency response plot of the transfer function of a system. It consists of two graphs. One is the plot of magnitude of sinusoidal transfer function versus log ω. The other is a plot of the phase angle of a sinusoidal function versus log ω.

40. What are the main advantages of Bode plot ?

Ans The main advantages are
(i) A simple method for sketching an approximate log curve is available.
(ii) Multiplication of magnitude can be done into addition.

41. Define corner frequency ?

Ans The frequency at which two asymptotic meet in a magnitude plot is called corner frequency.

42. Define phase lag and phase lead ?

Ans → A negative phase angle is called phase lag.
→ A positive phase angle is called phase lead.

43. What are M circles ?

Ans The magnitude of closed loop transfer function with unit feedback can be shown to be ~~same~~ for every value is M. The circles are called M-circles.

44. What are N-circles ?

Ans If the phase of closed loop transfer function with unity feedback is α, then $\tan\alpha$ will be in the form of circles for every value of α, these circles are called N-circles.

45. What is Nichols chart ?

Ans The chart consisting of M and N loci in the log magnitude versus phase diagram is called Nichols chart.

46. What is Nyquist ~~os~~ contour ?

Ans The contour that encloses entire right half of s-plane is called Nyquist contour.

~~47. State Nyquist stability criterion ?~~

~~Ans~~ 47. What are the two segments of Nyquist contour ?

Ans (i) A finite line segment c1 along the imaginary axis.

(ii) An arc c2 of infinite radius.

48. What are the effects of adding zero to a system ?

Ans Adding a zero to a system results in early peak to system response, thereby the peak overshoot increases appreciably.

49. What is the necessary condition for stability ?

Ans The necessary condition for stability is that all the coefficients of the characteristic polynomial is positive.

50. What is the necessary and sufficient condition for stability?

Ans The necessary and sufficient condition for stability is that all of the elements in the first column of the Routh Array should be positive.

51. Define relative stability?

Ans Relative stability is the degree of closeness of the system and indicates the strength or degree of stability.

52. What is Root Loci?

Ans The path taken by the roots of the open loop transfer function where the loop gain is varried from 0 to α are called root loci.

53. What are the main significance of root locus?

Ans (i) The main root locus technique is used for stability analysis.

(ii) Using root locus technique the range of values of "K", for a stable system can be determined"

54. What are the characterstics of Negative feedback?

Ans Negative feedback in a control system has following characterstics

(i) Reduction in the gain with better stability of the system.

(ii) Rejection of distrubance in the system.

(iii) High Accuracy.

control system

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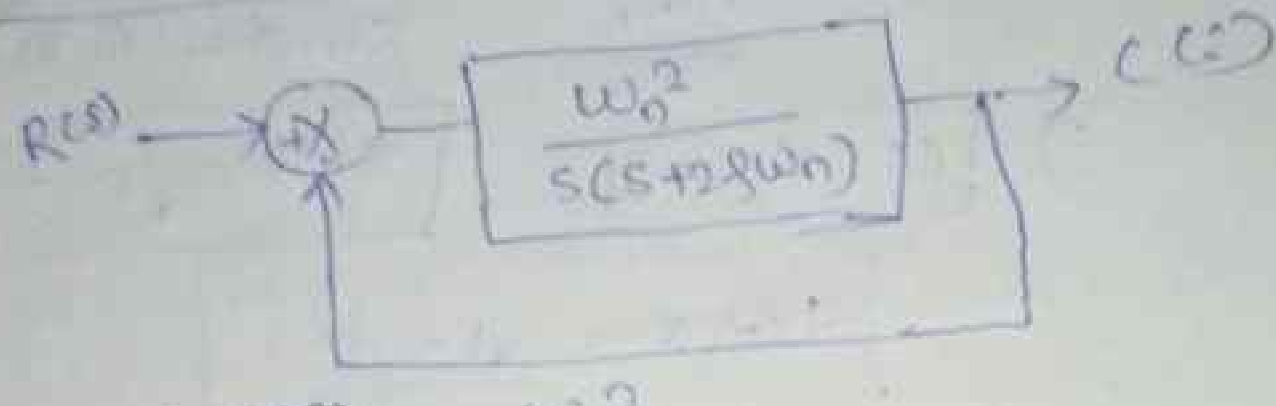
Time Response analysis

closed loop second order system



$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

So, the location of pole is

$$s^2+2\xi\omega_n s+\omega_n^2 = 0$$

$$\boxed{s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}}$$

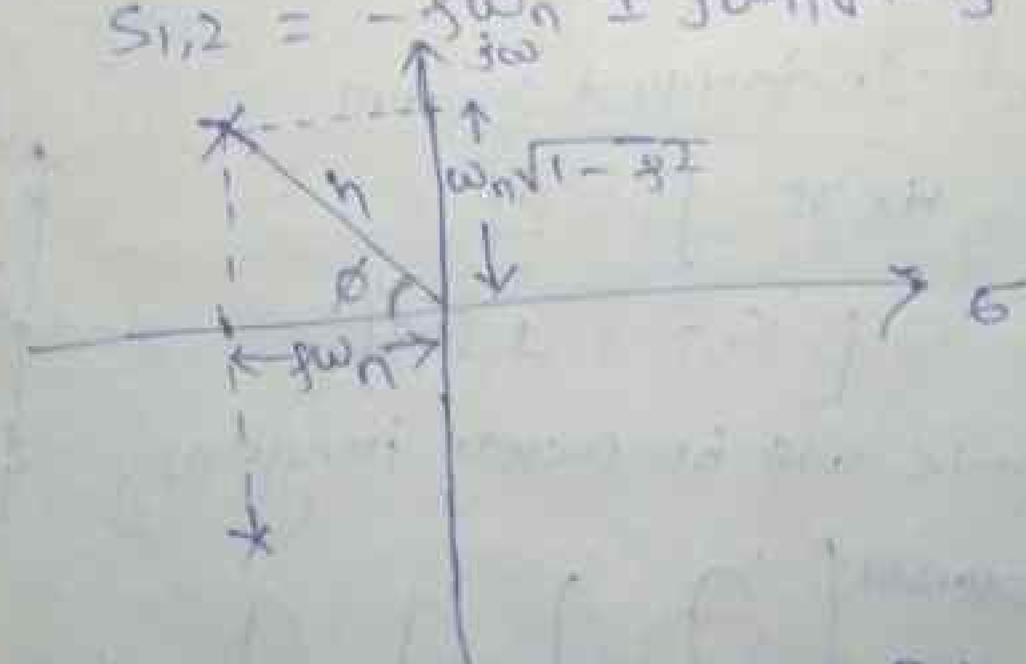
where ξ = damping ratio, ω_n = Natural frequency

Case-I

Underdamped system

In this case $\boxed{0<\xi<1}$

$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



Here, as real part of pole is $\xi\omega_n$

$$\therefore \boxed{\text{Time constant}(\tau) = \frac{1}{\xi\omega_n}}$$

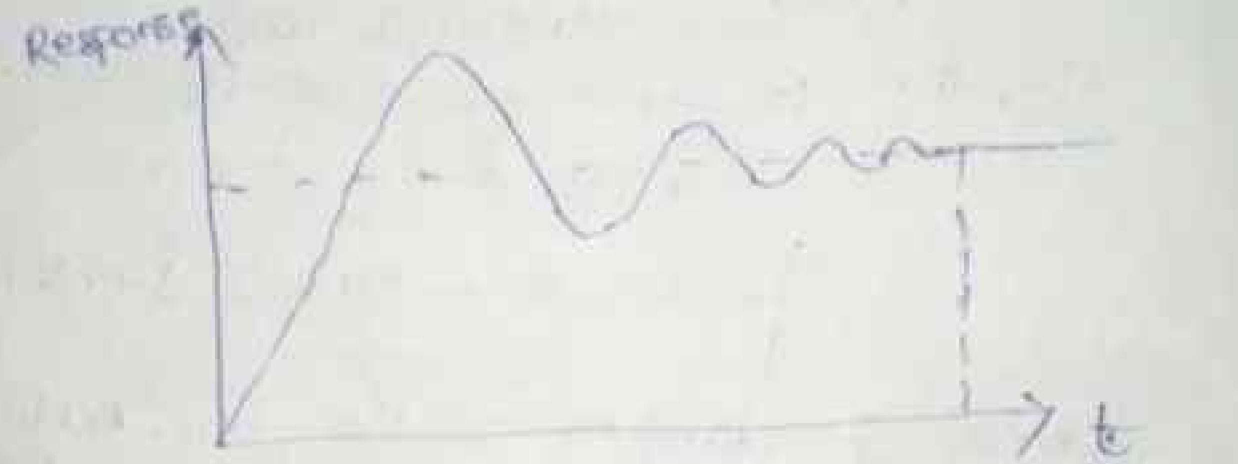
Now, $h = \sqrt{\omega_n^2(1-\xi^2) + \xi^2\omega_n^2}$

$\therefore$ ~~[illegible]~~ $\boxed{h = \omega_n}$

Now, $\cos\phi = \frac{\xi\omega_n}{\omega_n}$, $\sin\phi = \omega_n\sqrt{1-\xi^2}$

$\Rightarrow \boxed{\cos\phi = \xi}$ $\boxed{\sin\phi = \sqrt{1-\xi^2}}$

$$\boxed{\tan\phi = \frac{\sqrt{1-\xi^2}}{\xi}}$$



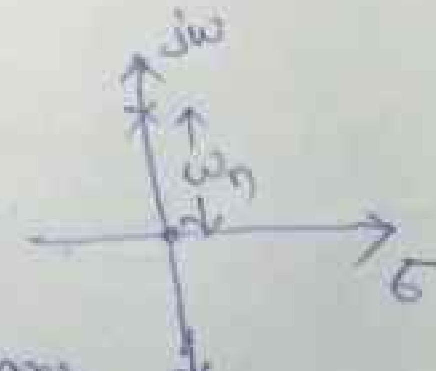
$t_s = 4\tau = \frac{4}{\xi\omega_n}$

$$\boxed{\text{Settling time } (t_s) = 4\tau = \frac{4}{\xi\omega_n}}$$

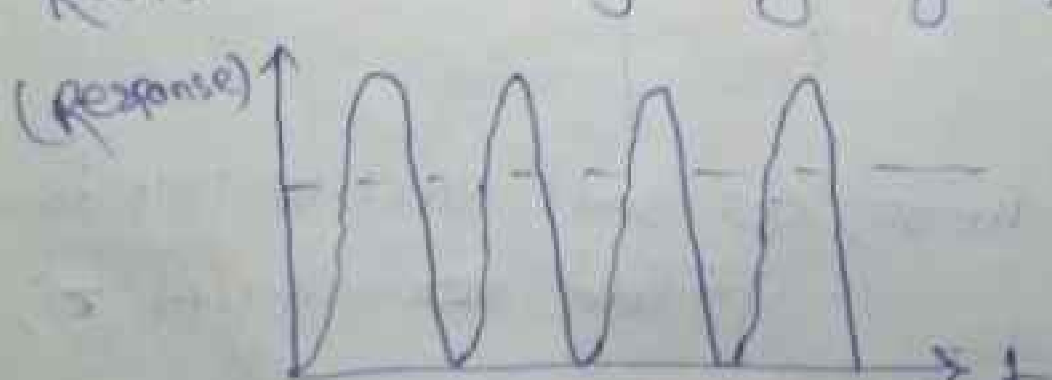
Case-II Undamped system

Here $\boxed{\xi = 0}$

$\boxed{s_{1,2} = \pm j\omega_n}$



Roots will be purely imaginary



$\boxed{\tau = \infty}$

Case-III critical damped system (3)

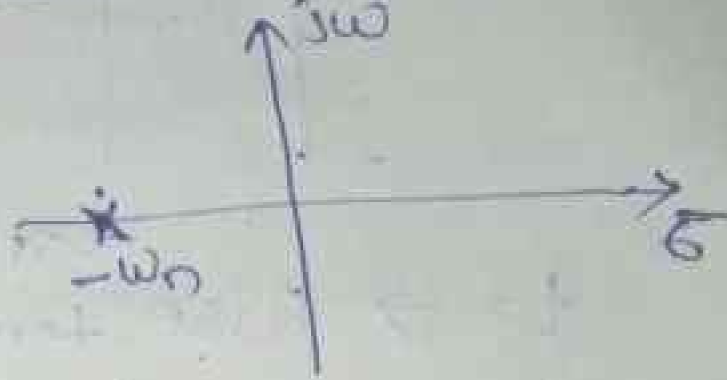
$\boxed{\text{Here } \xi = 1}$

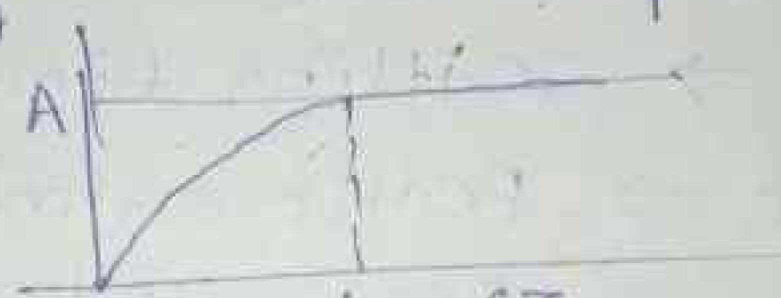
$$\boxed{s_{1,2} = -\omega_n}$$

Roots are purely real and equal

$$\boxed{\tau = \frac{1}{\omega_n}}$$

$$\boxed{t_s = 6\tau}$$





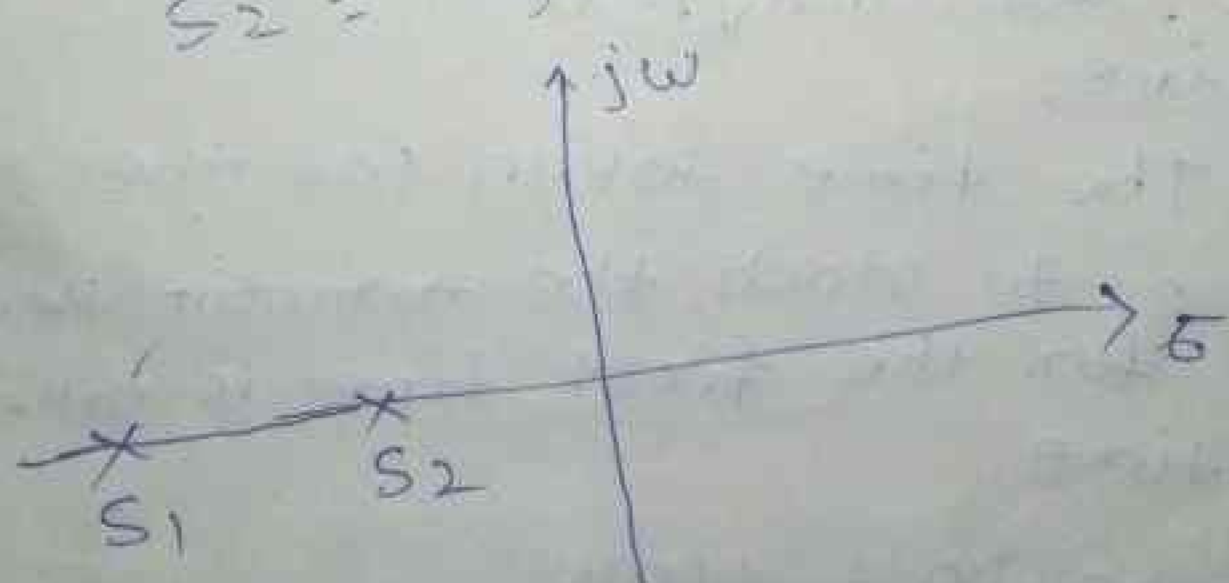
Case-IV Overdamped system

Here $\boxed{\xi > 1}$

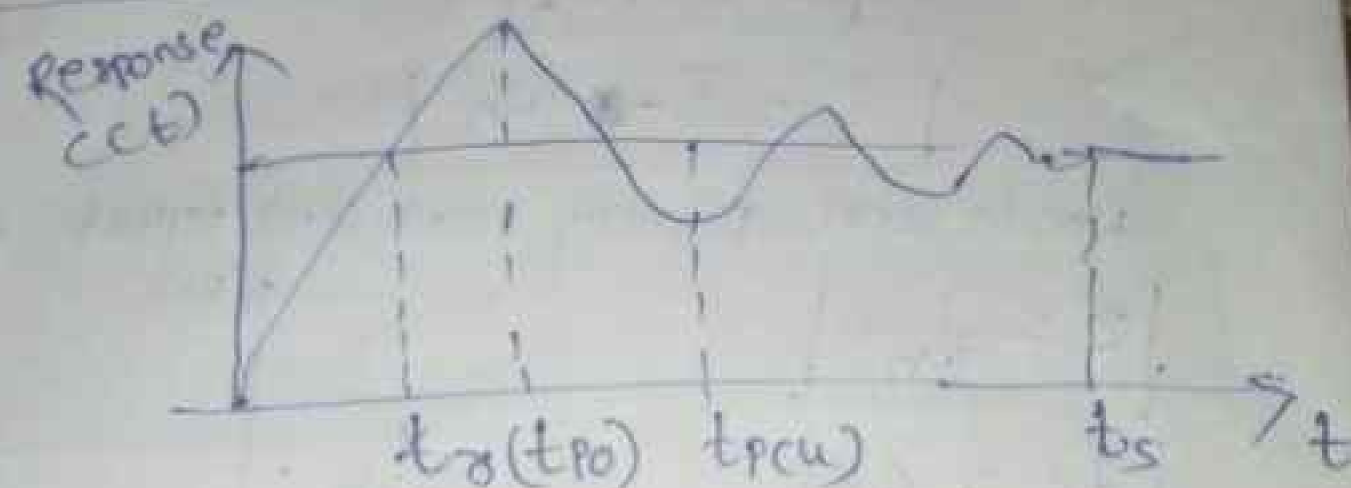
$$s_{1,2} = -\xi\omega_n \mp \omega_n\sqrt{\xi^2 - 1}$$

$$s_1 = -\xi\omega_n - \omega_n\sqrt{\xi^2 - 1}$$

$$s_2 = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$



2nd order underdamped system (9)



$t_r \rightarrow$ Rise time
$t_s \rightarrow$ settling time
$t_{po} \rightarrow$ peak overshoot time
$t_{pu} \rightarrow$ peak undershoot time

Rise time

It is defined as the time at which output will reach from 0% to 100% of the desired output.

$$\text{Rise time } (t_r) = \frac{\pi - \phi}{\omega_d} = \frac{\pi - \phi}{\omega_n\sqrt{1-\xi^2}}$$

$\omega_d \rightarrow$ damping frequency

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

Peak time

The time taken for the response to reach the ~~maximum~~ peak value for the first time is called peak time.

→ Peak overshoot time

$$t_{po} = \frac{(2n-1)\cdot\pi}{\omega_d} \quad \text{where } n = 1, 2, 3 \ldots$$

→ peak undershoot time

$$t_{pu} = \frac{(2n)\pi}{\omega_d}, \quad n = 0, 1, 2 \ldots$$

percentage overshoot

→ It exists at peak overshoot time.

→ It will exist in each oscillation.

$$\% M_{po} = \frac{c(t)|_{max} - c(t)|_{Desired}}{c(t)|_{Desired}} \times 100\,\%$$

$$\cancel{\% M_{pd} = e^{-\pi\delta/}}$$

$$\% M_{po} = e^{\frac{-(2n-1)\pi\delta}{\sqrt{1-\delta^2}}} \times 100\,\%$$

for first maximum overshoot

$$\% M_{po1} = e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}} \times 100\,\%$$

percentage Undershoot

→ It exist at peak undershoot time.

→ It is periodic in nature.

$$\% M_{pu} = \frac{c(t)|_{Desired} - c(t)|_{min}}{c(t)|_{Desired}} \times 100\,\%$$

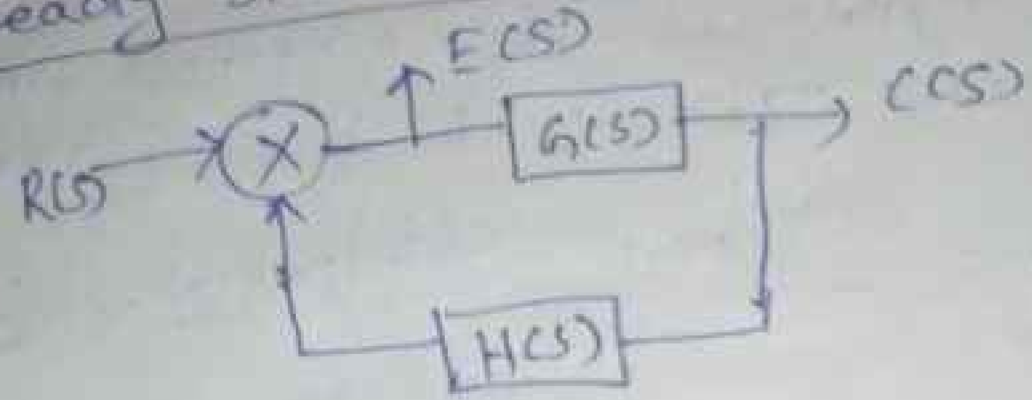
$$\% M_{pu} = e^{\frac{-(2n)\pi\delta}{\sqrt{1-\delta^2}}} \times 100\,\%$$

settling time

It is defined as the time taken by the response to reach final value and stay within specified error.

$$\boxed{t_s = 4\tau = \frac{4}{\xi\omega_n}}$$

steady state error



$$E(S) = \frac{R(S)}{1+G(S)\cdot H(S)}$$

$$\lim_{t\to\infty} e(t) = \lim_{S\to 0} S\cdot E(S)$$

$$= \lim_{S\to 0} \frac{S\cdot R(S)}{1+G(S)\cdot H(S)}$$

Case-I For step Input

$$R(S) = \frac{A}{S}$$

$$e_{ss} = \lim_{S\to 0} \frac{A\times S\times \frac{1}{S}}{1+G(S)\cdot H(S)}$$

$$= \frac{A}{1+\lim_{S\to 0}\{G(S)\cdot H(S)\}}$$

$$\boxed{e_{ss} = \frac{A}{1+K_p}}$$

K_p = positional error constant

$$K_p = \lim_{S\to 0} G(S)\cdot H(S)$$

case-II For Ramp Input

$$R(S) = \frac{A}{S^2}$$

$$e_{ss} = \lim_{s\to 0} \frac{s \cdot A/s^2}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{A/s}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{A}{s+s\cdot G(s)H(s)}$$

$$\therefore \boxed{e_{ss} = \frac{A}{K_v}}$$

$K_v \rightarrow$ velocity error constant

$$K_v = \lim_{s\to 0} s\cdot G(s)H(s)$$

<u>case-III</u> <u>For Parabolic input</u>

$$R(s) = A/s^3$$

$$e_{ss} = \lim_{s\to 0} \frac{s\cdot A/s^3}{1+G(s)\cdot H(s)}$$

$$= \lim_{s\to 0} \frac{A/s^2}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{A}{s^2+s^2G(s)H(s)}$$

$$= \frac{A}{0+\lim_{s\to 0}\{s^2G(s)H(s)\}}$$

$$\boxed{e_{ss} = \frac{A}{K_a}}$$

K_a = Acceleration error constant

$$K_a = \lim_{s\to 0} s^2G(s)H(s)$$

Type and order of the system

Type → It is defined as the total number of poles locating at origin.

order → It is defined as total number of poles in a system.

→ Type and order of a system is defined by its open loop transfer function.

* Type '0' system

Example $G(S)H(S) = \frac{K(ST_1+1)(ST_2+1)}{(ST_3+1)(ST_4+1)}$

Here K = DC gain of the system

For step Input

$$K_p = \lim_{S\to 0} G(S)H(S) = K \text{ (DC gain)}$$

$$\boxed{K_p = K = \text{DC gain}}$$

$$\boxed{e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+K} = \text{Finite}}$$

For Ramp input

$$K_v = \lim_{S\to 0} S\cdot G(S)H(S)$$

$$= \lim_{S\to 0} \frac{S\times(ST_1+1)(ST_2+1)}{(ST_3+1)(ST_4+1)}$$

$$\boxed{K_v = 0}, \quad \boxed{e_{ss} = \infty}$$

For Parabolic input

$$K_a = \lim_{S\to 0} S^2 G(S)H(S)$$

$$\Rightarrow \boxed{K_a = 0}$$

$$\boxed{e_{ss} = \infty}$$

* Type-1 system

Example $G(s)H(s) = \frac{K(sT_1+1)(sT_3+1)}{s(sT_2+1)(sT_4+1)}$

order = 3, Type = 1

for step input

$K_p = \lim_{s\to 0} G(s)H(s) = \frac{1}{0}$

$\Rightarrow \boxed{K_p = \infty}$ $\boxed{e_{ss} = 0}$

for Ramp input

$K_v = \lim_{s\to 0} s.G(s)H(s) = K$

$\Rightarrow \boxed{K_v = K = \text{DC gain}}$ $\boxed{e_{ss} = \frac{A}{K}}$

for Parabolic input

$K_a = \lim_{s\to 0} s^2 G(s)H(s) = 0$

$\Rightarrow \boxed{K_a = 0}$, $\boxed{e_{ss} = \infty}$

* Type-2 system

Example $G(s)H(s) = \frac{K(sT_1+1)(sT_2+1)}{s^2(sT_3+1)(sT_4+1)}$

for step input.

$K_p = \lim_{s\to 0} G(s)H(s)$

$\Rightarrow \boxed{K_p = \infty}$, $\boxed{e_{ss} = 0}$

for Ramp input

$K_v = \lim_{s\to 0} s.G(s)H(s) =$

$\Rightarrow \boxed{K_v = \infty}$, $\boxed{e_{ss} = 0}$

for Parabolic input

$K_a = \lim_{s\to 0} s^2 G(s)H(s)$

$\Rightarrow \boxed{K_a = K = \text{DC gain}}$

$\boxed{e_{ss} = \frac{A}{K_a} = \frac{A}{K}}$

→ Increasing type of a system will reduce steady state error but it also reduces stability of the system.

CONTROLLER

Needs of controller

→ The controller is provided to modify the error signal for better control action.

Different types of controller

→ (i) proportional controller
(ii) PI controller
(iii) PD controller
(iv) PID controller

proportional controller

→ It is a device that produces a control signal which is proportional to the input error signal.

proportional derivative controller

→ PD controller is proportional plus derivative controller which produces an output signal consisting of two times - one proportional to error signal and other proportional to the derivative of the ~~sg~~ signal.

significance of Integral controller

→ The proportional controller stabilizes the gain but produces a steady state error. The integral controller reduces the steady state error.

→ Disadvantage of proportional controller
(11) - The disadvantage of proportional controller is that it produces a constant steady state error.

→ Effect of PD controller is to increase the damping ratio of the system and so peak overshoot is reduced.

→ Effect of PI controller is to increase the order of the system by one, which results in reducing the steady state error.

STABILITY
(ROUTH-HURWITZ STABILITY)

→ Stability is defined in terms of two parameters.
(i) Absolute stability
(ii) Relative stability

Absolute Stability
It is defined in terms of ~~pole~~ location of pole and for closed loop system to be stable, all pole should be lie in the left half of plane.
- Routh-Hurwitz criteria, Root locus, Nyquist plot define absolute stability.
~~Relative st~~
Relative Stability
- It is defined in terms of damping ratio, Gain margin and phase margin.
If open loop system is non minimum phase system, then for closed loop

system to be stable, Gain margin
(12) and phase margin should be negative.

→ If open loop system is minimum phase system, then for closed loop system to be stable, Gain margin and phase margin should be positive.

→ In R-H criteria for stability, R-H table gives information about poles lying in the right half of S-plane. It will not give any information about poles lying on left half of s-plane.

Non minimum phase system

It is a system which contains at least one pole or zero on right half of S-plane.

Minimum phase system

It is a system which does not contain any pole or zero in the right half of S-plane.

Properties of Routh Table

(i) R-H criteria is applicable only for closed loop system because in formation of Routh Table, we use characteristic equation and characteristic equation is defined for closed loop system.

(ii) For closed loop system to be stable, all elements in first column of Routh table must be of same sign, either positive or negative. No. of times ~~sign~~ change in sign represents the number of poles lying on right half of s-plane, thus the system will be unstable.

(iii) In characteristic equation contains either is any power of 's' is missing that represents the presence of atleast one pole on right half and closed loop system will be unstable.

(iv) If characteristic equation contains either even power of 's' or odd power of 's', then there will be possible that poles can locate on imaginary axis.

(v) In characteristic equation, the co-efficient of 's' should be real, it should not be either imaginary, complex or sinusoidal.

Case - I

Any element of first column becomes zero. If any element of first column becomes 0, in that case, all element of next row will become ∞ and R-H table will terminate at that row. To extend R-H table, we will replace 0 with some variable, after that we will complete ~~our~~ Routh table and after completion of Routh table again, we will replace that variable with '0' in entire table.

Case - II All elements of any odd row becomes zero (0). If all elements of any odd row become 0, then auxiliary characteristic equation of even row just above that odd row will be purely

divisible with main characteristic equation and main characteristic equation will
(14) contain all pole of Auxilary characteristic equation and poles of Auxilary equation will locate at image location about either x or y axis.

→ In normal case, Routh table gives only information about poles, but for these cases, Routh table gives the location of poles.
→ The Auxilary characteristic equation will give exact location of image pole.
→ If more than one time odd row is becoming zero (0), then that represents the existence of more than one pair of pole at common image location and system will be unstable.

Examples

(1) The characteristic equation of a system is given as

$$s^4 + 2s^3 + 8s^2 + 4s + 3 = 0$$

Find the stability of the system by using R-H stability criteria.

s^4	1	8	3	
s^3	2	4	0	s^2 Row
s^2	6	3	0	$\frac{2\times8-1\times4}{2}$
s^1	3	0		$\frac{2\times3-1\times0}{2}$
s^0	3			

No sign changes in the first column. So no root in the right half.

All the poles will locate in left half side.
$\therefore$ system is stable.

(2) The characteristic equation of a system is given as
$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$. find the stability ?

s^5	1	2	3
s^4	1	2	5
s^3	$\boxed{0}\ d$	-2	0
s^2	$\frac{2d+2}{d}\ \boxed{+\infty}$	5	
s^1	[illegible]		

s^5	1	2	3
s^4	1	2	5
s^3	$\boxed{0}\ d$	-2	0
s^2	$\frac{2d+2}{d}\ \boxed{+\infty}$	5	
s^1	$-2-\frac{5d^2}{2d+2}\ \boxed{-2}$		
s^0	5		

Number of times sign change is two, so two pole will locate in right half side.
$\therefore$ system is unstable.

(3) The characteristic equation of a system is given as

(16) $s^6+3s^5+4s^4+6s^3+5s^2+3s+2=0$

Comment on stability of the system?

s^6	1	4	5	2
s^5	3	6	3	
s^4	2	4	2	
s^3	$0 \to 8$	$0 \to 8$		
s^2	2	2		
s^1	$0 \to 4$			
s^0	2			

Since all the element of s^3 row is zero. So Auxilary eq^n is

$A(s) = 2s^4+4s^2+2=0$

$\frac{dA(s)}{ds} = 8s^3+8s$

Finding solution of $A(s)$

$2(s^2+1)(s^2+1)=0$

$\Rightarrow s = \pm j1, \pm j1$

To find remaining pole divide $q(s)$ with $A(s)$

$$2s^4+4s^2+2 \overline{\big)\, s^6+3s^5+4s^4+6s^3+5s^2+3s+2 \,}\Big(\frac{s^2}{2}+\frac{3s}{2}+1$$

$$\underline{\overset{(-)}{s^6} \quad \overset{(-)}{+2s^4}}$$

$$3s^5+2s^4+6s^3+4s^2+3s+2$$

$$\underline{3s^5 \quad +6s^3 \quad +3s}$$

$$2s^4+4s^2+2$$

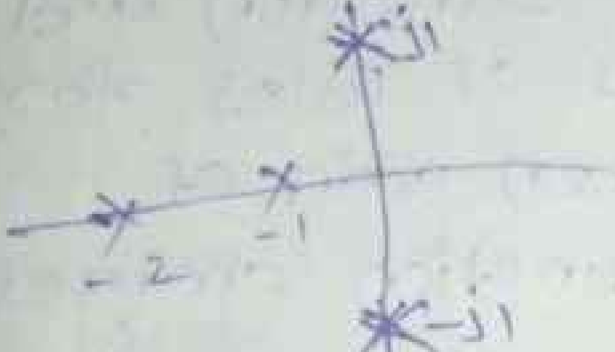
$$\underline{2s^4+4s^2+2}$$

$$0$$

$2s^4+4s^2+2=0$ and $s^2+3s+2=0$

$\Rightarrow s=\pm j1, \pm j1$ $\Rightarrow (s+1)(s+2)=0$

$\Rightarrow s=-1,-2$



system is unstable.

(4) The characteristic equation of a system is given as $s^4+5s^3+2s^2+5s+K=0$. Determine the range of 'K' for the system to be stable?

s^4	1	2	K
s^3	5	5	
s^2	1	K	
s^1	5-5K		
s^0	K		

$K>0$

$5-5K>0 \Rightarrow$

$5>5K \Rightarrow \frac{5}{5}>K$

$\Rightarrow K<1$

So $\boxed{0<K<1}$

— x —

ROOT LOCUS

(18)

→ In case of root locus, the variable is Gain of Transfer function and we calculate the locus of poles for closed loop system when gain of open loop transfer function changes.

→ So, for root locus, the stability is defined in terms of location of pole, Hence root locus defines absolute stability.

→ Different terms related to Root locus are

* Origination point
Root Locus will originate either from open loop pole or infinity but priority will be given to open loop pole and value of 'K' at origination point will be zero.

* Termination point
Root locus will terminate either at open loop zero or Infinity but priority will be given to open loop zero and value of 'K' at termination point will be "∞".

* Existence of Root locus on Real Axis
- Root locus will exist only at that location of real axis whose overall angle subtended by that location with open loop pole and zero is odd integral multiple of 180° (or) in other words Root locus will exist only in that section of real axis which contains odd number of

open loop poles and zero towards
(19) its right side.

* Existence of Root locus on complex plane

- Root locus will exist only at that location on complex plane whose overall subtended angle from all open loop poles and zeros is odd integral multiple of 180° (or) in other words, we substitute the given complex location in characteristic equation and then we will calculate the value of 'K'.
- If 'K' is real and positive, for given complex location, then that location is valid location and closed loop pole will exist at that location.
- If 'K' is either imaginary, complex or negative, then that location will be invalid location and closed loop pole will exist at that location.

* Break Point

Break point is location in s-plane where two poles coinsides simultaneously. It is of two types.

(i) Break away point
(ii) Break In point.

Breakaway point

It is a point in s-plane where the value of 'K' is maximum. Thus whenever Root locus shifts from real axis to complex plane always breakaway point will exist. To calculate location of breakaway point, we will differentiate 'K' in characteristic equation with respect to S and then

(2) set $\frac{dK}{ds} = 0$

- so breakaway point is maximum value of 'K' for root locus to be on real axis. If 'K' exceeds the maximum value, then root locus will enter into complex plane. Thus whenever root locus shifts from real axis to complex plane, always breakaway point exists. To calculate location of break away, we do

$$\frac{dK}{ds} = 0$$

Break in Point

- It is the minimum value of 'K' for the root locus to be on the real axis. when 'K' is less than this minimum value, then root locus will remain in complex plane. Thus, whenever root locus shifts from complex plane to real axis, always break in point exists. To calculate break in point we do

$$\frac{dK}{ds} = 0$$

* Angle of Asymptotes

We will calculate angle of asymptote only for that branch of root locus which is either originating or terminating to or from infinity.

case-I ($P > Z$)

From 'p' number of open loop pole, p branches of root locus will originate and out of 'p' branches, 'Z' number of branches will terminate on 'Z' number of open loop zeros and remaining $(P-Z)$ number of branch will terminate at '∞'. We will calculate angle of asymptote only for $(P-Z)$ number of branches which will terminate at '∞'.

$$\boxed{\theta_K = \frac{(2K+1)180^\circ}{P-Z}}, \quad K = 0, 1, 2 \ldots$$

case-II ($Z > P$)

From 'p' number of open loop pole, 'p' branch of root locus will originate and all 'p' branch will terminate at 'p' number of zeros. For termination of remaining $(Z-P)$ number of branch, root locus will originate from infinite and we will calculate angle of asymptote only for those branches of root locus which are originating from '∞'.

$$\boxed{\theta_K = \frac{(2K+1)180^\circ}{Z-P}}, \quad K = 0, 1, 2 \ldots$$

* Centroid

It is the origin of asymptotic line. The centroid (X) will be

$$\boxed{X = \frac{\Sigma P - \Sigma Z}{P-Z}}$$

ΣP = sum of the location of open loop pole.

ΣZ = sum of the location of open loop zero.

(32) P = Total No. of open loop pole
Z = Total no. of open loop zero

Gain margin

$$\text{Gain Margin} = \frac{K_{\text{marginally stable}}}{K_{\text{Desired}}}$$

- To calculate K_{marg}, we will use R-H criteria. In R-H criteria, we will calculate that value of 'K' for which odd row becomes 0 and then we will substitute that value of 'K' in even row just above that odd row, then we will calculate location of poles. If poles are located at imaginary axis for this value of K, then that 'K' will be marginal. for closed loop system to be.

Angle of Deparature

We will calculate angle of Deparature only for that open loop pole which lie at complex conjugate location.

$$\boxed{\theta_d = 180^\circ - [\phi_p - \phi_z]}$$

ϕ_p = Total angle subtended by remaining pole towards that pole whose deparature angle to be calculated.

ϕ_z = Total angle subtended by all open loop zero, towards that complex pole whose deparature angle we are calculating.

Angle of Arrival

We calculate angle of arrival only for complex zero.

$$\boxed{\theta_a = 180^\circ - [\phi_z - \phi_p]}$$

ϕ_z = Total angle subtended by remaining zero towards that complex zero whose arrival angle we are to calculate.

ϕ_p = Total angle subtended by all open loop pole towards that zero whose arrival angle we are to calculate.

EXAMPLES

(1) The open loop transfer function of system is $G(s) \cdot H(s) = \frac{K}{s(s+4)}$, $0 < K < \infty$. Draw its root locus?

→ $s_{p1} = 0$, $s_{p2} = -4$

→ characteristic equation

$$1 + G(s) \cdot H(s) = 0$$

$$1 + \frac{K}{s(s+4)} = 0$$

$$\Rightarrow s^2 + 4s + K = 0$$

$$\Rightarrow K = -(s^2 + 4s)$$

$$\Rightarrow \frac{dK}{ds} = 2s + 4 = 0$$

$$\Rightarrow \boxed{s = -2} \rightarrow \text{break point}$$

→ Angle of asymptote

$$\theta_K = \frac{(2K+1) \cdot 180^\circ}{P - Z}$$

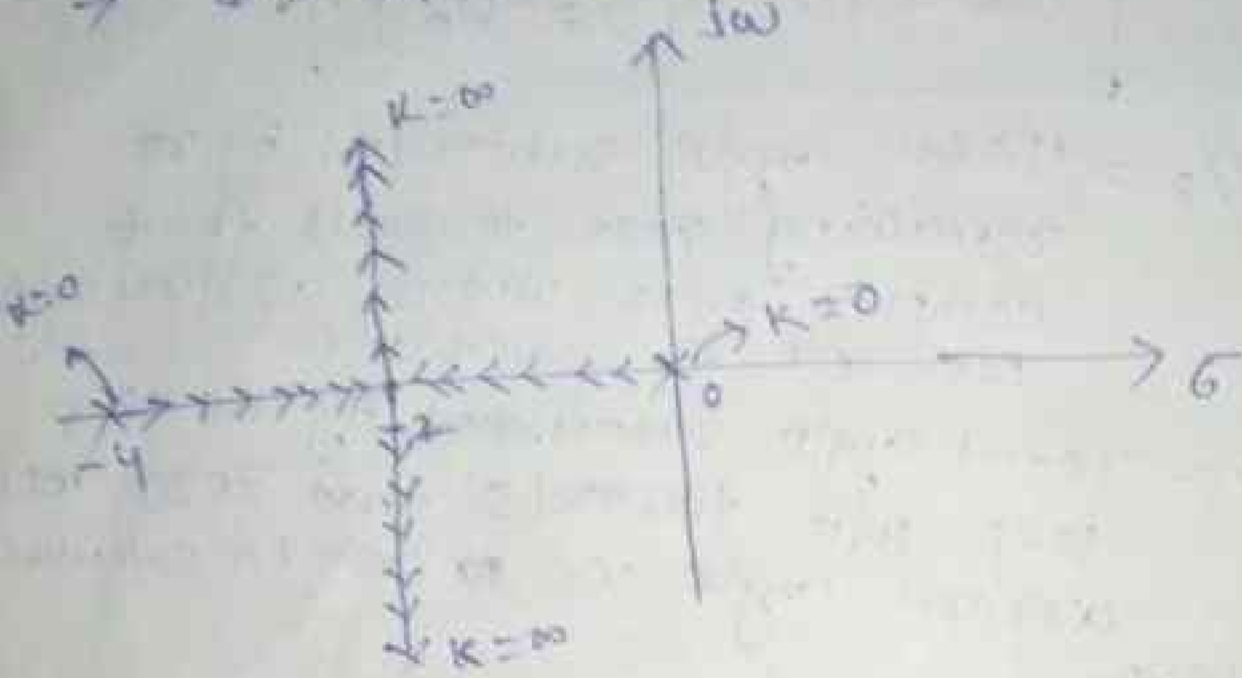
$$\theta_0 = 90^\circ$$

$$\theta_1 = 270^\circ$$

→ Centroid(x) = $\frac{\Sigma P - \Sigma Z}{P - Z} = \frac{-4-0}{2} = -2$

(2d)

→ ch. eqn = $s^2 + 4s$



(2) The open loop Transfer function of a system is given as

$G(s) \cdot H(s) = \frac{K}{s(s+1)(s+3)}$. Draw its root locus.

→ Sp = 0, -1, -3

→ characteristic equation

$1 + G(s) \cdot H(s) = 0$

$\Rightarrow 1 + \frac{K}{s(s+1)(s+3)} = 0$

$\Rightarrow K = -(s^2+s)(s+3)$

$\Rightarrow K = -(s^3 + 3s^2 + s^2 + 3s)$

$\Rightarrow K = -(s^3 + 4s^2 + 3s)$

$\Rightarrow \frac{dK}{ds} = 0$

$\Rightarrow \frac{d}{ds}(s^3 + 4s^2 + 3s) = 0$

$\Rightarrow 3s^2 + 8s + 3 = 0$

$\Rightarrow s = -0.45, -2.21$

→ Angle of asymptote

$$\theta_K = \frac{(2K+1)\cdot 180^\circ}{P-Z}$$

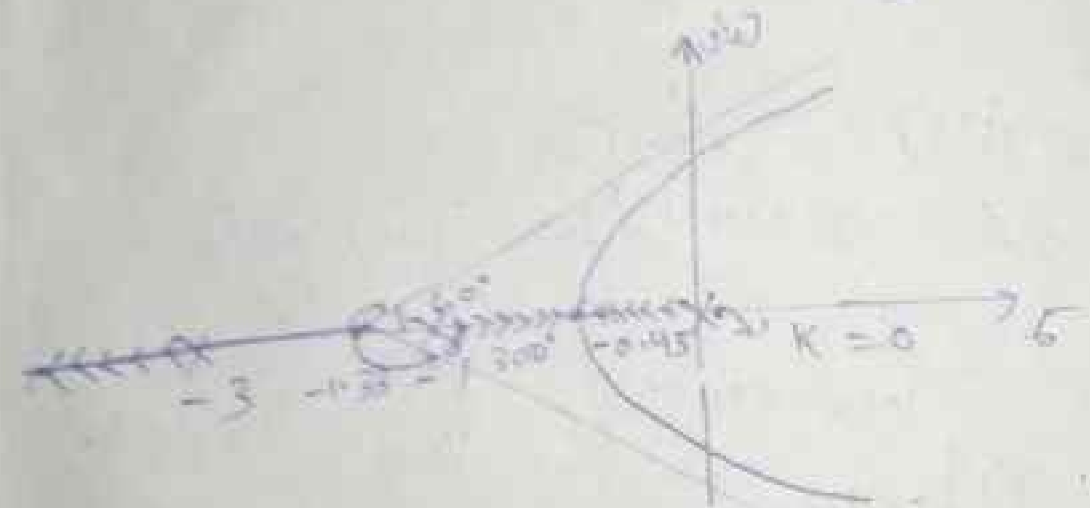
$\theta_0 = 60^\circ$

$\theta_1 = 180^\circ$

$\theta_2 = 300^\circ$

→ Centroid $(x) = \frac{\Sigma P - \Sigma Z}{P-Z}$

$= \frac{-4-0}{3} = \frac{-4}{3} = -1.33$



~~③ The OLTF of a system is~~

~~$G(s)\cdot H(s) = \frac{K}{s(s+4)(s^2+4s+13)}$. Draw its root locus.~~

~~→ $S_p = 0, -4$~~

~~→~~

NYQUIST PLOT

→ In Nyquist plot, the variable is entire s-plane, while in Bode plot variable is positive frequency line.

→ If open loop transfer function is minimum phase system, then for closed loop system to be stable, both Gain margin and Phase margin should be positive.

→ If open loop transfer function is non minimum phase system, then for closed loop system to be stable, both gain margin and phase margin should be negative.

→ Bode plot defines only Relative stability while Nyquist plot defines both absolute as well as Relative stability.

Example

$$G(s) \cdot H(s) = \frac{1}{s(s+1)}$$

Put $s = j\omega$, $G(s)H(s)\big|_{j\omega} = \frac{1}{j\omega(j\omega+1)}$

$$= \frac{1}{\omega\sqrt{1+\omega^2}} \angle -90^\circ - \tan^{-1}\omega$$

$$G(s)H(s)\big|_{s=j0} = \frac{1}{0}\angle -90^\circ = \infty \angle -90^\circ$$

$$G(s)H(s)\big|_{s=j1} = \frac{1}{\sqrt{2}} \angle -135^\circ$$

$$G(s)H(s)\big|_{s=j2} = \frac{1}{2\sqrt{5}} \angle -153^\circ$$